Journal of Stress Analysis
Vol. 5, No. 1, Spring − Summer 2020, 11-20

Bu-Ali Sina University

ORIGINAL RESEARCH PAPER

Calculation of Design Shape Sensitivity in Solid Mechanics Through a Novel Hybrid Method Using CVM and DSM

M. Sheikhi Azqandi[a,*], M. Hassanzadeh[b], M. Arjmand[a]

[a]Mechanical Engineering Department, Bozorgmehr University of Qaenat, Qaen, Iran.

[b]Mechanical Engineering Department, Gorgan Branch, Islamic Azad University, Gorgan, Iran.

Article info

Article history:
Received 13 April 2020
Received in revised form
13 July 2020
Accepted 01 August 2020

Keywords:
Hybrid method
Sensitivity analysis
Direct sensitivity method
Complex variables
Finite element

Abstract

In this study, a novel hybrid method was presented by considering the strengths and weaknesses of the two methods of the direct sensitivity method (DSM) and the complex variables method (CVM) and combining them to calculate shape sensitivity. The most of methods available are highly dependent on the values of step size variation related to the type of the problem. To validate the proposed method, some examples were analyzed by using the written finite element code. The comparison of results at solved problems indicated the independency of the proposed method from step size and only need to select an arbitrary small step size and the rounding error is negligible. It is a sign of its high computational performance which converges to reliable, stable, and high-precision results and saves calculation time compared to the other methods. The other advantages of the proposed method are the low volume of occupied memory and simplicity of implementation and its application in a wide range of engineering problems having simple and complicated equations.

Nomenclature

T	Absolute temperature	Y	Kinematic allowable temperature
T_0	Reference temperature	H^1	First-order Sobolev space
θ_0	Definite temperature	φ_i	Values vector of temperature
θ_∞	Ambient temperatures	M	Shape functions
n^i	Unit vector	B_T	Derivative matrix of shape functions
k	Heat conduction coefficient	K^{th}	Total thermal stiffness matrix
h	Convection heat transfer	φ	Temperature vector
q	Thermal flux vector	Q	Thermal load
g	Total number of elements	Ne	Inner thermal source
Γ_θ	Boundary conditions	Δx	Length of step siz
$\bar{\theta}$	Virtual temperature	u	Displacement
Ω	Domain		

*Corresponding author: M. Sheikhi Azqand (Assistant Professor)
E-mail address: mojtabasheikhi@buqaen.ac.ir
http://dx.doi.org/10.22084/jrstan.2020.21422.1142
ISSN: 2588-2597

1. Introduction

In the analysis and design of complex engineering problems, it is necessary to use methods to estimate how mechanical systems behave under the influence of desired or undesired changes of different variables. In other words, if a physical response is calculated from the mathematical model, the response sensitivity will be important compared to the other problem parameters. The technique to find the aforementioned sensitivity is named the sensitivity analysis which is employed to calculate the changes in the problem response to its parameters. The sensitivity information may be utilized to determine or distinguish the effect of uncertainties in the mathematical model and to predict the answer variations relative to the changes of problem parameters. Moreover, the sensitivity analysis can be utilized to optimally design a mechanical system with the help of the first- and second-order optimization methods which need to calculate the derivative of objective functions and constraints [1]. In addition to the optimal design of problems, the sensitivity analysis has a wide application in various fields including estimation of parameters, model simplification, data equalization, optimal control, uncertainty analysis, stochastic analyses, engineering reverse problems, topology optimization [2], shape optimization [3-4], and experimental design. It should be mentioned that this analysis extracts valuable information in the aforementioned fields.

In a few last decades, many methods have been presented concerning the sensitivity analysis in different fields of science and engineering. Finite difference method (FDM), analytical methods including direct sensitivity method (DSM) and adjoint variable methods (AVM), complex variables method (CVM), and semi-analytical method (SAM) are of the presented methods in this field [5-6].

The analytical methods have a lot of advantages than the numerical ones. The sensitivity calculated by the analytical methods are precise and do not need to use the step size, so, they are independent of it. However, in most cases, it is difficult to implement analytical methods in the finite element analysis code because the calculation of stiffness matrix is not always analytically possible. These derivatives are analytically calculated in the analytical method while it is difficult to calculate them in most cases, especially for derivative with respect to the geometry control parameters.

One of the most applicable numerical methods to calculate derivatives of functions is the Taylor series expansion around the desired point [7]. It is worthy to note that calculations can be carried out in both real and complex spaces using this method. For the first time, the complex variables method was presented by Lyness and Moler [8]. Lyness used it to determine derivatives of some complicated functions [9]. Squire and Trapp [10] also determined derivatives of real functions by using CVM. Recently, various papers have been presented to reintroduce the complex variable method [11-15]. The advantage of CVM over the finite difference method is the slight effect of rounding error on final results. So, it is not as sensitive as small steps and is effective for general nonlinear functions. Unfortunately, the CVM, like the FDM, is highly computational and requires a complete solution for each design variable. The semi-analytical method has been proposed to balance accuracy, efficiency and easily implemented and has the precision of analytical methods. The purpose of sensitivity analysis by FEM is to calculate the derivatives of the stiffness matrix, the mass matrix, and the force vector relative to the design variables.

In the analytical method, these derivatives are calculated analytically (mathematical relations). But in many cases, especially for calculating the derivative relative to the geometric variables, the calculation of derivatives is difficult. In the semi-analytical method, derivatives of stiffness matrix, force vector, and mass matrix are calculated numerically such as finite difference method, but the final solution is done analytically. In this way, it is possible to easily implement the finite difference method and accurately arrive at acceptable results. In this case, for the finite difference method, the possibility of cutting and rounding errors is possible and care must be taken in choosing the step size [16].

In recent years, the tendency to employ the sensitivity analysis for large-scale systems whose governing equations are the type of partial derivatives differential ones has increased. One of these, the optimal design of mechanical systems, has been at the center of interest using the calculation of sensitivity by considering temperature constraints [17-21].

Lots of research works have been published about heat transfer sensitivity in mechanical systems thus far. For instance, Haftka and Malkus [22] presented the sensitivity analysis of conduction heat transfer based on a discrete model including two linear and nonlinear modes. Dems [18] also carried out developing the sensitivity analysis of the size and shape of design parameters for the problem of nonlinear conduction heat transfer in two states of steady and transient. Furthermore, Meric [23] presented a shape sensitivity analysis for the problem of steady-state conduction heat transfer. Tortorelli et al. [24] investigated the linear and nonlinear sensitivity analysis of conduction heat transfer based on a continuous model. Chen and Tong [25] studied the sensitivity analysis in functionally graded material (FGM) in steady and transient states. Using semi-analytical method (SAM), Fernandez, and Tortorelli [26] performed the sensitivity analysis in steady, transient, and dynamic mode problems. Besides, Furuta et al. [27] examined the sensitivity analysis of heat transfer problems in nanoscale dimensions. Lee [28] scrutinized the sensitivity analysis in two-dimension heat transfer problems in inhomogeneous objects via the CVM as well. Established upon different environmental criteria like heat transfer initiated from the sunlight, Silva and Ghisi [29] predicted how the performance of structures is using the sensitivity analysis.

Direct sensitivity method is a very accurate and fast method, however, due to the difficulty of implementing it analytically and its non-generality for all issues, this method was generally combined with the finite difference method. This would reduce the accuracy and uncertainty of the semi-analytical method because the results are generally dependent on the step size. The complex variables method is an accurate numerical method, and independent of the step size, but being time-consuming is a disadvantage. In the current study, by considering the strengths and weaknesses of these two methods, a novel method to calculate the sensitivity analysis in heat transfer problems was developed through simultaneous using DSM and CVM as well as combining them. The results obtained by the proposed method are independent of step size and have high precision. In the next section, modeling and obtaining the equations of finite element and analysis of heat transfer equations, and then employing the proposed method to analyze the sensitivity for a few problems having heat transfer are presented. Finally, the validation of the proposed method is investigated.

2. Primary Equations

To model and obtain the equations governing to heat transfer problems, a homogeneous and isotropic three-dimensional solid material is considered as shown in Fig. 1. The steady thermal conduction equation and boundary conditions are in the form of relations (1) [30]:

$$\begin{aligned} -k\theta_{,ii} &= g && @\ \Omega \\ \theta &= \theta_0 && @\ \Gamma_\theta^0 \\ k\theta_{,i}n^i &= q && @\ \Gamma_\theta^1 \\ k\theta_{,i}n^i + h(\theta-\theta_\infty) &= 0 && @\ \Gamma_\theta^2 \end{aligned} \tag{1}$$

where $\theta = T - T_0$ that T is the absolute temperature, T_0 is the reference temperature in a material (body) stress-free mode, and θ_0 is the definite temperature of material, θ_∞ is the ambient temperature, n^i is the i component of unit vector perpendicular to the boundary, k is the heat conduction coefficient of the material, h is the convection heat transfer of the material, q is the thermal flux vector, g is the inner thermal source, and Γ_θ^0, Γ_θ^1, and Γ_θ^2 are the boundary with definite temperature, thermal flux, and convection heat in that order respectively. It should be mentioned that the superscripts and subscripts indicate the component and derivative concerning the field variables, respectively. Furthermore, the Einstein summation convention was utilized for duplicate index all through this paper.

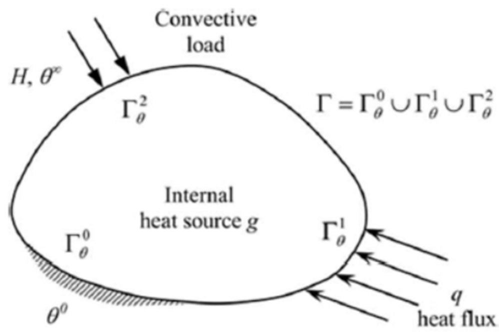

Fig. 1. Heat transfer problem modeling for three-dimensional solid material [16].

The weak form of the heat transfer problem will be obtained if we multiply both sides of Eq. (1) by the virtual temperature $\bar{\theta}$ and then integrate on the domain Ω.

$$\begin{aligned} &\int_\Omega k\theta_{,i}\bar{\theta}_{,i}d\Omega + \int_{\Gamma_\theta^2} h\theta\bar{\theta}d\Gamma \\ &= \int_\Omega g\bar{\theta}d\Omega + \int_{\Gamma_\theta^1} q\bar{\theta}d\Gamma + \int_{\Gamma_\theta^2} h\theta_\infty\bar{\theta}d\Gamma \end{aligned} \tag{2}$$

Eq. (2) for all $\bar{\theta} \in Y$ in which Y is the kinematic allowable temperature space is in the form of Eq. (3).

$$Y = \left\{\theta \in [H^1(\Omega)] :\ \theta = 0,\ x \in \Gamma_\theta^0\right\} \tag{3}$$

In Eq. (3), the parameter H^1 is the first-order Sobolev space. Eq. (4) presents the energy bilinear and load linear forms for the temperatures.

$$\begin{aligned} A(\theta,\bar{\theta}) &\equiv \int_\Omega k\theta_{,i}\bar{\theta}_{,i}d\Omega + \int_{\Gamma_\theta^2} h\theta\bar{\theta}d\Gamma \\ L(\bar{\theta}) &\equiv \int_\Omega g\bar{\theta}d\Omega + \int_{\Gamma_\theta^1} q\bar{\theta}d\Gamma + \int_{\Gamma_\theta^2} h\theta_\infty\bar{\theta}d\Gamma \end{aligned} \tag{4}$$

Therefore, Eq. (2) is simply rewritten as follows:

$$A(\theta,\bar{\theta}) = L(\bar{\theta}) \quad \text{for all } \theta \in Y \tag{5}$$

3. Finite Element Modeling

The equations governing the heat transfer problems presented in the previous section are solved using FEM. With the assumption of having continuous values of θ, they can be calculated into an element by interpolation of node values in the form of Eq. (6):

$$\theta = M_i\varphi_i \tag{6}$$

where φ_i is the values vector of temperature at element nodes and M is a matrix consisting of corresponding shape functions of each node. Temperature gradients can be calculated in the form of Eqs. (7) where B_T

is the derivative matrix of shape functions concerning position variables and is defined as Eq. (8).

$$\nabla\theta = \begin{Bmatrix} \dfrac{\partial\theta}{\partial x} \\ \dfrac{\partial\theta}{\partial y} \end{Bmatrix} = B_T\varphi \tag{7}$$

$$B_T = \begin{Bmatrix} \dfrac{\partial M}{\partial x} \\ \dfrac{\partial M}{\partial y} \end{Bmatrix} \tag{8}$$

By substituting Eqs. (6) to (8) in Eq. (2) and simplifying them, stiffness matrix, and force vector can be calculated in FEM.

$$\bar{\varphi}^T K^{th}\varphi = \bar{\varphi}^T Q \tag{9}$$

where

$$\begin{aligned} K^{th} &= \int_\Omega kB_T^T B_T d\Omega + \int_{\Gamma_\theta^T} hM^T M d\Gamma \\ Q &= \int_\Omega gM^T d\Omega + \int_{\Gamma_\theta^1} qM^T d\Gamma + \int_{\Gamma_\theta^2} h\theta_\infty M^T d\Gamma \end{aligned} \tag{10}$$

Since Eq. (9) should be established for all $\bar{\varphi}$, Eq. (11) can be concluded as:

$$K^{th}\varphi = Q \tag{11}$$

where K^{th} is the total thermal stiffness matrix which is acquired from superposing the stiffness matrixes of elements and temperature vector coefficients of initiated from convection boundary condition, and Q is the thermal load resulted from heat transfer processes including thermal flux.

To calculate the variables K^{th} and Q, Eqs. (12) and (13) should be used on all available elements in the domain investigated in the material.

$$K^{th} = \sum_{e=1}^{Ne}\left(\int_{\Omega^e} kB_T^T B_T d\Omega + \int_{\Gamma_\theta^{2e}} hM^T M d\Gamma\right) \tag{12}$$

$$Q = \sum_{e=1}^{Ne}\Bigg(\int_{\Omega^e} gM^T\Omega + \int_{\Gamma_\theta^{1e}} qM^T M d\Gamma + \int_{\Gamma_\theta^{2e}} h\theta_\infty M^T M d\Gamma\Bigg) \tag{13}$$

In the above relations, Ne is the total number of elements, and integrals are taken on the elements or their boundaries. After assembling and obtaining the stiffness matrix as well as the total force vector of material, temperature values are acquired in the whole material by applying boundary conditions and then solving Eq. (5).

4. Shape Design Sensitivity Analysis

As mentioned earlier, the finite element equation is obtained through Eq. (11) for the steady-state thermal conduction problem. In this paper, it is assumed that the parameters k, h, g, and q are independent of temperature.

DSM is based on the implicit derivation of balance (equilibrium) equations. Thus, the derivative of Eq. (11) is taken concerning the design parameters h_p and $p = 1, , P$ in the analysis of sensitivity for temperature. The following term is obtained for the sensitivities $\dfrac{\partial\varphi}{\partial h_p}$ by ordering the taken derivative.

$$K^{th}\frac{\partial\varphi}{\partial h_p} = -\frac{\partial K^{th}}{\partial h_p}\varphi + \frac{\partial Q}{\partial h_p} \tag{14}$$

The recent equation is similar to Eq. (11) and just the right side of the aforementioned equation named as heat quasi-load vector should be calculated. When the right-side values of the new terms are determined, $\dfrac{\partial\varphi}{\partial h_p}$ can also be calculated by solving the system of Eq. (14).

The terms $\dfrac{\partial\varphi}{\partial h_p}$ and $\dfrac{\partial K^{th}}{\partial h_p}$ can be precisely obtained in the analytical method while it is difficult and intolerable to exactly determine them. Moreover, how to acquire the aforementioned terms may vary from a problem to another. In the traditional semi analytical method (TSAM), these terms are calculated by FDM and the sensitivities are then obtained using solving the system of Eq. (14). Simplicity is one of the advantages of SAM, but its weakness is being sensitive to the change of step size and the results are under the influence of rounding and cutting errors. This weakness is more apparent especially when the number of terms is high and the presence of error in their calculation leads to creating a bigger total error. However, in the current research, the complex variable was utilized to numerically calculate the mentioned terms. This leads to eliminating the dependency of results on the step size. The other advantage of the proposed method is lower occupied memory because complex numbers are not used in the whole implemented computer code, so, the code runtime and calculation volumes become lower. Besides, this method results in converging to precise and secure results in lower time.

In this paper, the finite element method was used to analyze the heat transfer problems and sensitivity analysis was implemented in the finite element code. In general, sensitivity analysis by the analytical method is performed with two ways:
First, to derive from the heat transfer equation and then solve it numerically (for example, the finite element method).
Second, the problem is first formulated numerically (for

example, by the finite element method), and then derive from the discretized equation. In this paper, the second way is used, which was the same as the DSM.

5. Complex Variable Derivative Method

As mentioned earlier, CVM is based on the Taylor series expansion so that it takes a complex step on the imaginary axes. To extract FDM approximation to calculate derivatives, it can be developed Taylor series at point x using Newton forward, and backward steps and the below formulation is then obtained by subtracting them.

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} \tag{15}$$

Relation (15) has a second-order error to calculate the derivative in which Δx is the length of change step. The weaknesses of this method are its high calculation time and the inaccurate possibility of derivative values of functions. The first one initiates from requiring Eq. (15) to two solutions having good convergence for calculation of functions. The second one is because of being sensitive derivatives to step size. To minimize the rounding error of numbers, the step size should be selected small [7]. It should be mentioned that too small step size may result in producing an error that eliminates meaningful numbers. In order to create a balance, the optimal value for step size is not previously determined and may be varied from a function and/or a design variable to another.

If the function Taylor series is expanded using a complex step, Eq. (16) will be obtained [7].

$$\begin{aligned} f(x+i\Delta x) = f(x) + i\Delta x\frac{df}{dx} - \frac{\Delta x^2}{2}\frac{d^2 f}{dx^2} \\ -\frac{i\Delta x^3}{6}\frac{d^3 f}{dx^3} + \frac{\Delta x^4}{24}\frac{d^4 f}{dx^4} - \cdots \end{aligned} \tag{16}$$

By separating two real and imaginary parts of Eq. (16) and solving the imaginary one, derivative relation is accessible as Eq. (17).

$$\frac{df}{dx} \approx \frac{\mathrm{Im}[f(x+i\Delta x)]}{\Delta x} \tag{17}$$

Similar to relation (15), relation (17) has a second-order error for calculation of derivative $O(2)$. Therefore, the function and its derivative are acquired without subtraction error by calculating the function with a complex argument. Hence, it can be expressed that the real part is the value of the function.

Since the problem is linear here, the variables of stiffness matrix K^{th} and force vector Q are not as a function of temperature. As a result, Eqs. (18) and (19) can be employed to calculate the terms of $\frac{\partial Q}{\partial h_p}$ and $\frac{\partial K^{th}}{\partial h_p}$.

$$\frac{\partial K^{th}}{\partial h_p} = \frac{\mathrm{Im}[K^{th}(h_p + i\Delta h_p)]}{\Delta h_p} \tag{18}$$

$$\frac{\partial Q}{\partial h_p} = \frac{\mathrm{Im}[Q(h_p + i\Delta h_p)]}{\Delta h_p} \tag{19}$$

After calculation of the above terms, the values of sensitivity analysis of the desired parameters with respect to the problem variables, i.e. $\frac{\partial Q}{\partial h_p}$, are calculated through Eq. (20).

$$\frac{\partial \varphi}{\partial h_p} = (K^{th})^{-1}\left(-\frac{\partial K^{th}}{\partial h_p}\varphi + \frac{\partial Q}{\partial h_p}\right) \tag{20}$$

When the parameters get sensitive to displacement and temperatures, the sensitivity of the desired functions will be calculable based on the following discussion.

If the response R is a function of design parameters h_p and $p = 1, \cdots, P$, also, dependent on the displacement (u) and temperature (φ) fields, it will be as Eq. (21). The derivative of response R concerning the design parameter h_p is defined as Eq. (22).

$$R = R(\varphi(h_p), h_p) \tag{21}$$

$$\frac{dR}{dh_p} = \frac{\partial R}{\partial h_p} + \frac{\partial R}{\partial \varphi}\frac{\partial \varphi}{\partial h_p} \tag{22}$$

$\frac{dR}{dh_p}$ is also obtained using Eq. (23):

$$\frac{dR}{dh_p} = \frac{\mathrm{Im}[\phi(\varphi + i\Delta\varphi, h_p + i\Delta h_p)]}{\Delta h_p} \tag{23}$$

In which:

$$\Delta\varphi = \frac{\partial \varphi}{\partial h_p}\Delta h_p \tag{24}$$

In this research work, to analyze the problems, a 2D finite element code with the aforementioned formulation and assumptions was written in MATLAB. This code utilizes linear shape functions for the four-node square element. The equations of shape functions of these elements can be extracted from most reference books of finite element [31]. The geometry of and meshing problems are made in ABAQUS and then MATLAB was applied to analyze them. To solve the system of equations, the MATLAB's solver was employed. In fact, in this paper, problem analysis was done by using of written code and ABAQUS software was used to produce geometry. A small step should be created in an appropriate node coordinates to change in desire geometry variables; of course, it should be in the form of imaginary. To this end, the first node located in the desired parameter is chosen. The partial disturbance for X and Y coordinates is dependent on the sensitivity which should be calculated. All of the other nodes remain unchanged. If the goal of the problem is to determine the sensitivity analysis concerning material characteristics like Young modulus, a change step should be considered for the studied characteristic.

6. Numerical Examples

In this section, some problems are utilized to validate the proposed method for analyzing the sensitivity with the help of FEM. The results obtained for the sensitivity analysis are compared to the current research, TSAM, and FDM. The same code of problem without the sensitivity part is used for FDM as well.

6.1. Sensitivity Analysis in a Thin Rectangular Plate

Consider a 10m side rectangular plate as shown in Fig. 2. The three sides of this plate were kept at $T = 100°$C. Sinusoidal temperature distribution was applied to the upper boundary of the plate. Hence, the boundary conditions governing the plate are in the form of Eqs. (25) and (26).

$$T(x,0) = T_1$$
$$T(0,y) = T_1 \tag{25}$$
$$T(W,y) = T_1$$

$$T(x,H) = T_m \sin\left(\frac{\pi x}{W}\right) + T_1 \tag{26}$$

In Eq. (26), the parameter T_m is the domain of the sinusoidal function and equal to 100°C. In this problem, solving the equations having partial derivatives using separating the variables results in solving the Laplace equation. The temperature distribution in the whole plate will be in the form of Eq. (27) [32].

$$T = T_m \frac{\sinh\left(\frac{\pi y}{W}\right)}{\sinh\left(\frac{\pi H}{W}\right)} \sin\left(\frac{\pi x}{W}\right) + T_1 \tag{27}$$

By deriving Eq. (27) concerning one of the sides of the square (H), the sensitivity $\frac{dT}{dH}$ is calculable as Eq. (28). It should be paid attention that given the asymmetry available in the boundary condition of the plate, the sensitivity values of temperature will be different compared to the other side of the square (W).

$$\frac{dT}{dH} = -\frac{\pi T_m}{W} \sinh\left(\frac{\pi y}{W}\right) \sin\left(\frac{\pi x}{W}\right) \frac{\cosh\left(\frac{\pi H}{W}\right)}{\left(\sinh\left(\frac{\pi H}{W}\right)\right)^2} \tag{28}$$

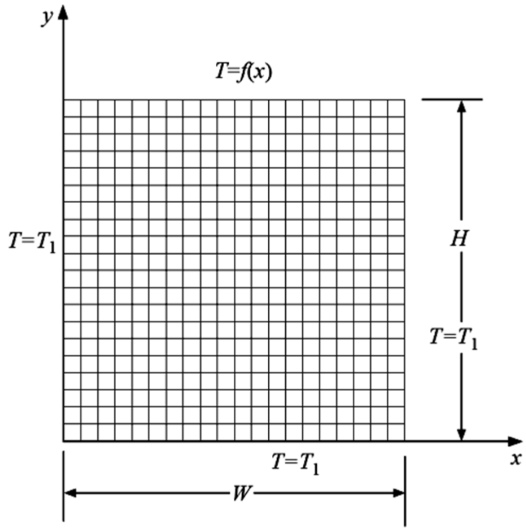

Fig. 2. The geometry shape of a thin rectangular plate under heat transfer and boundary conditions governing it.

To numerically analyze the problem by FEM, each side is divided into 20 equal parts so that the whole plate consists of 400 four-node square elements.

The temperature sensitivity is presented in terms of plate length (H) in Table 1 using SAM and FDM at the center of a point with the coordination of $x = \frac{W}{2}$ and $y = \frac{H}{2}$. Fig. 3 shows a comparison between the values of temperature sensitivity analysis with respect to the plate length based on the logarithm values of step size. As can be seen, FDM has no reliable convergence in all step sizes. The correct performance is observable only at a limited interval from the step size of $10^{-4} - 10^{-9}$. The divergence of problem is apparent and the accuracy of sensitivity analysis is eliminated, especially at the length of the steps much smaller than 10^{-12}. For instance, the relative error value during calculating the sensitivity analysis is equal to 126% at the step length of 10^{-15}. It is worthy to mention that employing FDM is not possible at the length of the steps smaller than 10^{-15}. Despite FDM, the method proposed in the current research has a reliable convergence as well as an acceptable accuracy in the calculation of sensitivity analysis even the length of the small steps. The difference amount between the results obtained at the step length values of 10^{-1} and 10^{-15} for the proposed method is equal to 0°C while this difference at FDM is 7.93°C which is almost more than 100% error.

It can be concluded again that FDM is sensitive to step size while the proposed semi-analytical method eliminates its dependency on step size by employing a complex variable. Moreover, for all step sizes in this method, the relative error value of the sensitivity analysis compared to the real one is very negligible and equal to 0.07%. Furthermore, the code runtime and calculations volume of the proposed method are also lower than that of FDM.

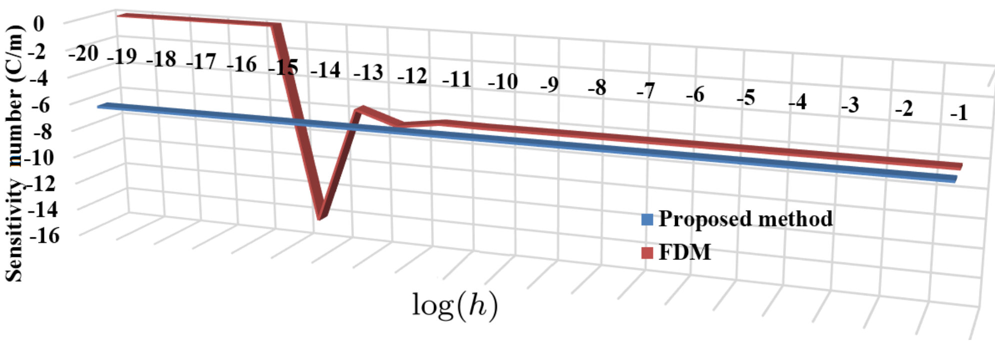

Fig. 3. The comparison between the values of temperature sensitivity analysis with respect to plate length.

Table 1
Temperature sensitivity analysis on a rectangular plate relative to the length of one side $\left(\frac{dT}{dH}\right)$.

log(h) (m)	FDM (°C/m)	Proposed method (°C/m)
-1	-6.280701577318	-6.27950984450222
-2	-6.279521762882	-6.27950984450222
-3	-6.279509963711	-6.27950984450222
-4	-6.279509844092	-6.27950984450222
-5	-6.279509834428	-6.27950984450222
-6	-6.279509790375	-6.27950984450222
-7	-6.279508966145	-6.27950984450222
-8	-6.279525877062	-6.27950984450222
-9	-6.279570641254	-6.27950984450222
-10	-6.279918807195	-6.27950984450222
-11	-6.285461040534	-6.27950984450222
-12	-6.245670647331	-6.27950984450222
-13	-6.679101716145	-6.27950984450222
-14	-5.684341886081	-6.27950984450222
-15	-14.210854715202	-6.27950984450222
-16	0.000000000000	-6.27950984450222
-17	0.000000000000	-6.27950984450222
-18	0.000000000000	-6.27950984450222
-19	0.000000000000	-6.27950984450222
-20	0.000000000000	-6.27950984450222
Elapsed time (s)	11.513062	7.965710
	Exact sensitivity	6.28371254

6.2. *Calculation of Shape Sensitivity in a Long Cylindrical Tube*

In this section, a cylindrical tube with $r_i = 0.4$m (inner radius) and $r_o = 1$m (outer radius) was considered as shown in Fig. 4. There was no heat generation source into the tube and its heat conduction coefficient was equal to 0.2W/mK. The boundary conditions governing this problem on the inner and outer surfaces of the tube are in the form of Eq. (29).

$$
\begin{aligned}
r = r_i &\Longrightarrow\ T = 500\text{K} \\
r = r_o &\Longrightarrow\ q = -20\text{J/m}
\end{aligned}
\tag{29}
$$

In this problem, the goal is to calculate the derivative of the tube temperature concerning the outer radius $\left(\frac{dT(r)}{dr_0}\right)$. To validate the result of the written code, the solution of the heat transfer problem is illustrated using the written code and ABAQUS analysis in Figs. (5) and (6), respectively. The similarity of the temperature distribution results in the tube is clear by comparing two aforesaid figures.

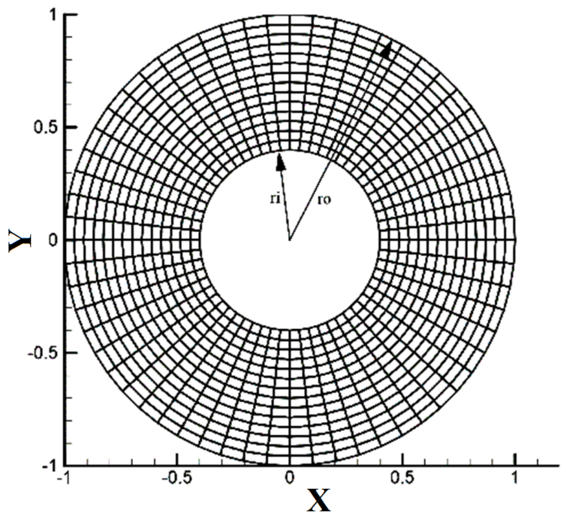

Fig. 4. The geometry shape of a long cylindrical tube under heat transfer.

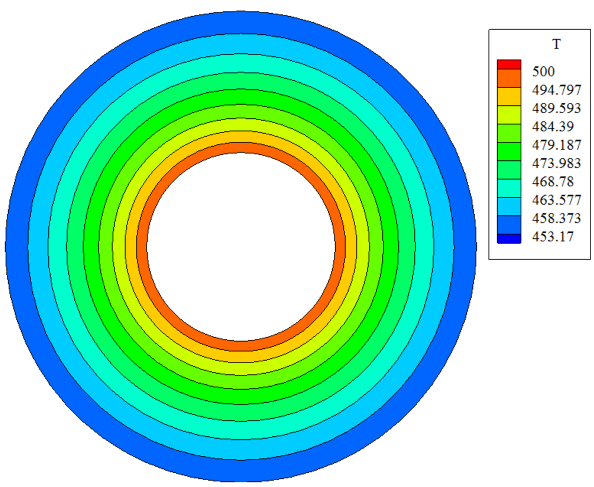

Fig. 5. Temperature distribution in the long cylindrical tube by the proposed code.

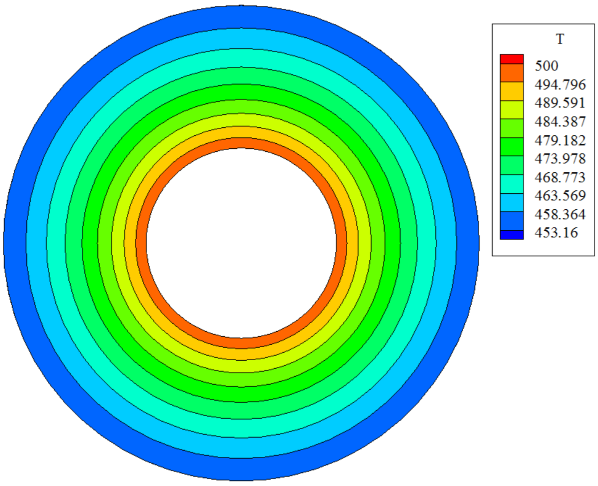

Fig. 6. Temperature distribution in the long cylindrical tube by ABAQUS.

Table 2 indicates a comparison between the results of the sensitivity analysis obtained from the proposed method, SAM, FDM, and TSAM.

The results of sensitivity analysis for the three aforementioned methods are plotted in Fig. 7. As can be seen, the results of FDM and TSAM have an error at the step size smaller than 10^{-13}, but there is a stable convergence in the proposed method at the step size smaller than 10^{-3}. It is clear that FDM and TSAM are sensitive to step size but there is no sensitivity to it for the combination of SAM with CVM. The relative difference amount between the value of sensitivity analysis of Table 2 at two step sizes of 10^{-3} and 10^{-16} are equal to 2.8%, 19440%, and 2725% for the proposed method, FDM, and TSAM, respectively. In addition, the calculation time of the proposed method is also lower than that of FDM and TSAM.

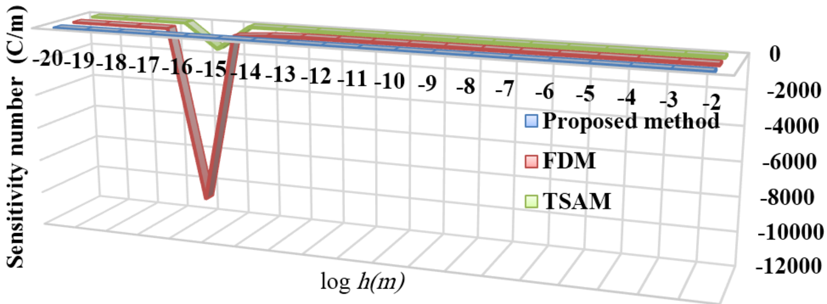

Fig. 7. The comparison between the values of temperature sensitivity analysis the long cylindrical tube.

Table 2
Temperature sensitivity analysis on the long cylindrical tube relative to outer radius $\left(\frac{dT}{dr_0}\right)$.

log(h) (m)	FDM (°C/m)	TSAM (°C/m)	Proposed method (°C/m)
-2	-52.3629756002094	-55.3766747803753	-49.6581215308640
-3	-52.3617390971083	-52.3902501429957	-52.3332341022271
-4	-52.3617267373311	-52.3620116810284	-52.3614415229236
-5	-52.3617266622978	-52.3617294525968	-52.3617237506746
-6	-52.3617273131549	-52.3617265904056	-52.3617265729905
-7	-52.3617234193807	-52.3617206030492	-52.3617266012352
-8	-52.3616762393431	-52.3617244238044	-52.3617266014823
-9	-52.3621679349162	-52.3615095477798	-52.3617266014774
-10	-52.3601784152561	-52.3611955601236	-52.3617266014771
-11	-52.3186827194876	-52.3726111210512	-52.3617266014668
-12	-53.1485966348555	-52.2944735451879	-52.3617266014927
-13	-55.7065504835919	-54.2073993718553	-52.3617266014828
-14	-119.3711796076970	-29.9294349775606	-52.3617266014960
-15	-255.7953848736360	-38.3982092519222	-52.3617266014813
-16	-10231.8153949454000	-1480.3636062773500	-52.3617266014945
-17	0.0000000000000	0.0000000000000	-52.3617266014871
-18	0.0000000000000	0.0000000000000	-52.3617266014999
-19	0.0000000000000	0.0000000000000	-52.3617266014829
-20	0.0000000000000	0.0000000000000	-52.3617266014731
Elapsed time (s)	4.16	2.79	2.40

7. Conclusions

By combining DSM and CVM, the current study presents a novel method to calculate the geometry sensitivities in heat transfer problems. DSM has good efficiency and saves calculation time compared to the other methods. This is because of calculating the sensitivities only in the required situations of material. However, the common methods utilize an exact approach to derive stiffness and mass matrixes, so, the derivation for each type of finite element can be tedious. To improve the efficiency of DSM, exact derivation can be replaced with the FDM numerical method. It should be noted that the common sensitivity calculation methods are highly dependent on the values of step size. Therefore, using the advantages of both DSM and CVM, which are not influenced by the step size, a novel computational method to calculate the sensitivities of heat transfer problems can be recommended. In this research, the superiority of the proposed method compared to the common computational methods was indicated by investigating some examples. In the current method, only if a small step size is selected, the rounding error is negligible, so that it provides a reliable, exact, and stable answer for the sensitivity in problems. Meanwhile, the calculation time is considerably low as well. The low volume of the occupied memory and simplicity of implementation in the form of the computer code is of the other advantages of this method. In the end, the error of the calculated sensitivities in the common computational methods can considerably decrease by the application of complex variables. The numerical calculation of sensitivity in the outline of DSM can be improved using the complex variables. In comparison with the common approaches, the proposed method has high precision and ensures good efficiency in the total range of step size.

Acknowledgment

The authors would like to acknowledge the financial support of Bozorgmehr University of Qaenat for this research.

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